Mathematical Reasoning

Question1

The negation of the statement "For every real number x; $x^2 + 5$ is positive" is

KCET 2024

Options:

A. For every real number x; $x^2 + 5$ is not positive.

B. For every real number x; $x^2 + 5$ is negative.

C. There exists atleast one real number x, such that $x^2 + 5$ is not positive.

D. There exists atleast one real number x, such that $x^2 + 5$ is positive.

Answer: C

Solution:

The negation of statement "For every real number x; $x^2 + 5$ is positive" is there exists at least one real number x such that $x^2 + 5$ is not positive.

Question2

The contrapositive of the statement.

"If two lines do not intersect in the same plane, then they are parallel." is

KCET 2023

Options:

A. If two lines are parallel, then they intersect in the same plane.

B. If two lines are not parallel, then they do not intersect in the same plane.

C. If two lines are parallel, then they do not intersect in the same plane.

D. If two lines are not parallel, then they intersect in the same plane.

Answer: D

Solution:

Given, statement

"If two lines do not intersect in the same plane, then they are parallel.

Contrapositive It two lines intersect in the same plane, then they are not parallel.

Converse If two lines do not intersect in the same plane, then they are parallel.

Question3

If $P(n) : 2^n < n!$ Then the smallest positive integer for which $P(n)$ is true if

KCET 2020

Options:

A. 2

B. 3

C. 4

D. 5

Answer: C

Solution:

We have,

$P(n) = 2^n < n!$
$P(1) = 2 < 1!$ False
$P(2) = 2^2 < 2\ !$ False
$P(3) = 2^3 < 3\ !$ False
$P(4) = 2^4 < 4\ !$ True

$\therefore$ The smallest position integer for which $P(n)$ is true for $n = 4$

Question4

The negation of the statement "For all real numbers x and $y, x + y = y + x$" is

KCET 2020

Options:

A. For all real numbers x and $y, x + y \neq y + x$

B. For some real numbers x and $y, x + y = y + x$

C. For some real numbers x and $y, x + y \neq y + x$

D. For some real numbers x and $y, x - y = y - x$

Answer: C

Solution:

The negation of the statement "For all real x and $y, x + y = y + x$" is

For some real number

x and $y, x + y = y + x$

Question5

The negative of the statement "All continuous functions are differentiable."

KCET 2019

Options:

A. Some continuous functions are not differentiable

B. All continuous functions are not differentiable

C. All differentiable function are continuous

D. Some continuous function are differentiable

Answer: A

Solution:

Some continuous function are not differentiable.

Question6

$P(n) : 2^{2n} - 1$ is divisible by k for all $n \in N''$ is true, then the value of ' k ' is

KCET 2018

Options:

A. 6

B. 3

C. 7

D. 2

Answer: B

Solution:

To determine the value of k such that $P(n) = 2^{2n} - 1$ is divisible by k for all $n \in \mathbb{N}$, we start by examining a specific case:

Consider $n = 1$:

$P(1) = 2^{2\times1} - 1 = 2^2 - 1 = 4 - 1 = 3$

Since $P(1) = 3$, it follows that $P(n)$ is divisible by 3.

Therefore, the value of k is 3.

Question7

The negation of the statement " 72 is divisible by 2 and $3''$ is

KCET 2018

Options:

A. 72 is not divisible by 2 or 72 is not divisible by 3

B. 72 is divisible by 2 or 72 is divisible by 3

C. 72 is divisible by 2 and 72 divisible by 3

D. 72 is not divisible by 2 and 3

Answer: A

Solution:

To find the negation of the statement "72 is divisible by 2 and 72 is divisible by 3," we can use De Morgan's law, which states:

$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$

Here, let:

P be "72 is divisible by 2"

Q be "72 is divisible by 3"

The negation becomes:

"72 is not divisible by 2 or 72 is not divisible by 3"

Thus, the negation is correctly expressed in Option A.

Key Points:

The original statement is a conjunction (using "and").

Its negation is a disjunction (using "or") of the individual negations.

So, the correct answer is:

Option A:

72 is not divisible by 2 or 72 is not divisible by 3.

Question8

The contrapositive statement of the statement "If x is prime number, then x is odd" is

KCET 2017

Options:

A. If x is not odd, then x is not a prime number

B. If x is a prime number, then x is not odd

C. If x is not a prime number, then x is not odd

D. If x is not a prime number, then x is odd

Answer: A

Solution:

The original statement is:

"If x is a prime number, then x is odd."

Let's denote:

P : x is a prime number

Q : x is odd

The contrapositive of an "if-then" statement "if P, then Q" is:

"if not Q, then not P."

So, for our case:

"Not Q" means: x is not odd (i.e., even).

"Not P" means: x is not a prime number.

Thus, the contrapositive becomes:

"If x is not odd, then x is not a prime number."

This corresponds to Option A.
